Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Monday 25 June 2012 − Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40109A

1. A particle P moves in a plane such that its position vector $\mathbf{r}$ metres at time t seconds ($t > 0$) satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} - \frac{2}{t}\mathbf{r} = 4\mathbf{i}.$$

When $t = 1$, the particle is at the point with position vector $(\mathbf{i} + \mathbf{j})$ m.

Find $\mathbf{r}$ in terms of t.

(9)

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2. A rocket, with initial mass 1500 kg, including 600 kg of fuel, is launched vertically upwards form rest. The rocket burns fuel at a rate of 15 k g s^{-1} and the burnt fuel is ejected vertically downwards with a speed of 1000 m s^{-1} relative to the rocket. At time t seconds after launch ($t \leq 40$) the rocket has mass m kg and velocity v m s^{-1}.

(*a*) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{1000}{m}\frac{\mathrm{d}m}{\mathrm{d}t} = -9.8.$$

(5)

(*b*) Find v at time t, $0 \leq t \leq 40$.

(5)

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3. A uniform rod PQ, of mass m and length $3a$, is free to rotate about a fixed smooth horizontal axis L, which passes through the end P of the rod and is perpendicular to the rod. The rod hangs at rest in equilibrium with Q vertically below P. One end of a light inextensible string of length $2a$ is attached to the rod at P and the other end is attached to a particle of mass $3m$. The particle is held with the string taut, and horizontal and perpendicular to L, and is then released. After colliding, the particle sticks to the rod forming a body B.

(*a*) Show that the moment of inertia of B about L is $15ma^2$.

(2)

(*b*) Show that B first comes to instantaneous rest after it has turned through an angle $\arccos \frac{9}{25}$.

(10)

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4. A body consists of a uniform plane circular disc, of radius r and mass $2m$, with a particle of mass $3m$ attached to the circumference of the disc at the point P.

The line PQ is a diameter of the disc. The body is free to rotate in a vertical plane about a fixed smooth horizontal axis, L, which is perpendicular to the plane of the disc and passes through Q. The body is held with QP making an angle β with the downward vertical through Q, where $\sin \beta = 0.25$, and released from rest. Find the magnitude of the component, perpendicular to PQ, of the force acting on the body at Q at the instant when it is released.

[*You may assume that the moment of inertia of the body about L is* $15mr^2$.]

(6)

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5. The points P and Q have position vectors $4\mathbf{i} - 6\mathbf{j} - 12\mathbf{k}$ and $2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ respectively, relative to a fixed origin O.

Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, act along $\overrightarrow{OP}$, $\overrightarrow{QO}$ and $\overrightarrow{QP}$ respectively, and have magnitudes 7 N, 3 N and $3\sqrt{10}$ N respectively.

(*a*) Express $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ in vector form.

(3)

(*b*) Show that the resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ is $(2\mathbf{i} - 10\mathbf{j} - 16\mathbf{k})$ N.

(2)

(*c*) Find a vector equation of the line of action of this resultant, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a parameter.

(5)

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6. A uniform circular pulley, of mass $4m$ and radius r, is free to rotate about a fixed smooth horizontal axis which passes through the centre of the pulley and is perpendicular to the plane of the pulley. A light inextensible string passes over the pulley and has a particle of mass $2m$ attached to one end of a particle of mass $3m$ attached to the other end. The particles hang with the string vertical and taut on each side of the pulley. The rim of the pulley is sufficiently rough to prevent the string slipping. The system is released from rest.

(*a*) Find the angular acceleration of the pulley.

(8)

When the angular speed of the pulley is Ω, the string breaks and a constant braking couple of magnitude G is applied to the pulley which brings it to rest.

(*b*) Find an expression for the angle turned through by the pulley from the instant when the string breaks to the instant when the pulley first comes to rest.

(4)

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7. (*a*) A uniform lamina of mass m is in the shape of a triangle ABC. The perpendicular distance of C from the line AB is h. Prove, using integration, that the moment of inertia of the lamina about AB is $\frac{1}{6}mh^2$.

(7)

(*b*) Deduce the radius of gyration of a uniform square lamina of side $2a$, about a diagonal.

(3)

The points X and Y are the mid-points of the sides RQ and RS respectively of a square $PQRS$ of side $2a$. A uniform lamina of mass M is in the shape of $PQXYS$.

(*c*) Show that the moment of inertia of this lamina about XY is $\frac{79}{84}Ma^2$.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\frac{d\mathbf{r}}{dt} - \frac{2}{t}\mathbf{r} = 4\mathbf{i}$	
	IF$= e^{\int \frac{-2}{t} dt} = \frac{1}{t^2}$	M1 A1
	$\frac{d}{dt}(\frac{\mathbf{r}}{t^2}) = \frac{1}{t^2}4\mathbf{i}$	M1 A1
	$\frac{\mathbf{r}}{t^2} = \int \frac{1}{t^2}4\mathbf{i}\ dt$	M1
	$= \frac{-1}{t}4\mathbf{i} + \mathbf{C}$ (**C** not needed for A1)	A1
	$\mathbf{r} = -4t\mathbf{i} + \mathbf{C}t^2$	
	$t = 1, \mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{i} + \mathbf{j} = -4\mathbf{i} + \mathbf{C} \Rightarrow 5\mathbf{i} + \mathbf{j} = \mathbf{C}$	M1 A1
	$\mathbf{r} = -4t\mathbf{i} + (5\mathbf{i} + \mathbf{j})t^2$	A1
		9
2. **(a)**	$(m+\delta m)(v+\delta v) - (-\delta m)(1000 - v) - mv = -mg\delta t$	M1 A2
	$\delta v + \frac{1000}{m}\delta m = -g\delta t$	
	$\frac{dv}{dt} + \frac{1000}{m}\frac{dm}{dt} = -9.8$ PRINTED ANSWER	DM1 A1
		(5)
(b)	$\frac{dv}{dt} - \frac{15000}{1500 - 15t} = -9.8$	M1
	$\frac{dv}{dt} - \frac{1000}{100 - t} = -9.8$	
	$v = \int_0^t \frac{1000}{100-t} - 9.8\ dt$	M1
	$= \left[-1000\ln(100-t) - 9.8t\right]_0^t$	A1
	$v = 1000\ln\frac{100}{(100-t)} - 9.8t$	DM1 A1
		(5) **10**

Question Number	Scheme	Marks
3.		
(a)	$I_P = \frac{4}{3}m(\frac{3a}{2})^2 + 3m(2a)^2 = 15ma^2$ PRINTED ANSWER OR $= \frac{1}{3}m(\frac{3a}{2})^2 + m(\frac{3a}{2})^2 + 3m(2a)^2 = 15ma^2$	M1 A1
		(2)
(b)	KE gain = PE loss	
	$\frac{1}{2}3mv^2 = 3mg.2a$ **OR** $\frac{1}{2}(12ma^2)\Omega^2 = 3mg.2a$	M1
	$v = 2\sqrt{ag}$ $\Omega = \sqrt{\frac{g}{a}}$	A1
	CAM: $3mv.2a = 15ma^2\omega$ **OR** CAM: $(12ma^2)\Omega = 15ma^2\omega$	M1 A1
	$\omega = \dfrac{2av}{5a^2} = \dfrac{4}{5}\sqrt{\dfrac{g}{a}}$	A1
	KE loss = PE gain	
	$\frac{1}{2}15ma^2\omega^2 = mg\dfrac{3a}{2}(1-\cos\theta) + 3mg.2a(1-\cos\theta)$	M1 A1 A1
	$\cos\theta = \dfrac{9}{25}$ i.e. $\theta = \cos^{-1}(\dfrac{9}{25})$ PRINTED ANSWER	M1 A1
		(10) **12**
4.	$M(Q)$, $2mgr\sin\beta + 3mg2r\sin\beta = 15mr^2\ddot{\theta}$	M1 A1
	OR $M(Q)$, $5mg\frac{8r}{5}\sin\beta = 15mr^2\ddot{\theta}$	M1 A1
	$(\swarrow)$ $2mg\sin\beta + 3mg\sin\beta - X = 2mr\ddot{\theta} + 3m2r\ddot{\theta}$	M1 A1
	OR $(\swarrow)$ $5mg\sin\beta - X = 5m\frac{8r}{5}\ddot{\theta}$	M1 A1
	solving for X,	M1
	$X = \dfrac{11mg}{60}$	A1
		6

Question Number	Scheme	Marks
5.		
(a)	$\mathbf{F}_1 = 7.\frac{1}{\sqrt{4^2+(-6)^2+(-12)^2}}\begin{pmatrix}4\\-6\\-12\end{pmatrix} = \begin{pmatrix}2\\-3\\-6\end{pmatrix}$	B1
	$\mathbf{F}_2 = 3.\frac{1}{\sqrt{2^2+4^2+4^2}}\begin{pmatrix}-2\\-4\\-4\end{pmatrix} = \begin{pmatrix}-1\\-2\\-2\end{pmatrix}$	B1
	$\mathbf{F}_3 = 3\sqrt{10}.\frac{1}{\sqrt{2^2+(-10)^2+(-16)^2}}\begin{pmatrix}2\\-10\\-16\end{pmatrix} = \begin{pmatrix}1\\-5\\-8\end{pmatrix}$	B1
		(3)
(b)	$\sum \mathbf{F}_\text{i} = \begin{pmatrix}2\\-3\\-6\end{pmatrix} + \begin{pmatrix}-1\\-2\\-2\end{pmatrix} + \begin{pmatrix}1\\-5\\-8\end{pmatrix} = \begin{pmatrix}2\\-10\\-16\end{pmatrix}$ PRINTED ANSWER	M1 A1
		(2)
(c)	Taking moments about *O*,	
	$\begin{pmatrix}4\\-6\\-12\end{pmatrix} \text{x} \begin{pmatrix}1\\-5\\-8\end{pmatrix} = \begin{pmatrix}x\\y\\z\end{pmatrix} \text{x} \begin{pmatrix}2\\-10\\-16\end{pmatrix}$	M1
	$\begin{pmatrix}-12\\20\\-14\end{pmatrix} = \begin{pmatrix}-16y+10z\\2z+16x\\-10x-2y\end{pmatrix}$ put $x=0 \Rightarrow z=10 \Rightarrow y=7$	A1 A1 M1
	so, $\mathbf{r} = \begin{pmatrix}0\\7\\10\end{pmatrix} + \lambda\begin{pmatrix}1\\-5\\-8\end{pmatrix}$ is a vector equation.	A1
		(5) **10**

Question Number	Scheme	Marks
6.		
(a)	$3mg - T_1 = 3mr\alpha$	M1 A1
	$T_2 - 2mg = 2mr\alpha$	M1 A1
	$r(T_1 - T_2) = \frac{1}{2}4mr^2\alpha$	M1 A1
	adding,	DM1
	$mg = 7mr\alpha$	
	$\alpha = \dfrac{g}{7r}$	A1
		(8)
(b)	$G = 2mr^2\beta$	M1 A1
	$0^2 = \Omega^2 - 2\beta\theta$	M1
	$\theta = \dfrac{mr^2\Omega^2}{G}$	A1
		(4)
		12

Question Number	Scheme	Marks
7.		
(a)	$\rho = \dfrac{2m}{bh}$	B1
	$\delta m = \rho \dfrac{b(h-x)}{h}\delta x$	M1
	$= \dfrac{2m}{h^2}(h-x)\delta x$	
	$\delta I = \dfrac{2m}{h^2}(h-x)x^2\delta x$	A1
	$I = \int_0^h \dfrac{2m}{h^2}(h-x)x^2 \mathrm{d}x = = \dfrac{2m}{h^2}\left[\dfrac{hx^3}{3} - \dfrac{x^4}{4}\right]_0^h$	M1 A1
	$= \dfrac{1}{6}mh^2$ PRINTED ANSWER	DM1 A1
		(7)
(b)	$I = 2$ x $\dfrac{1}{6}m(a\sqrt{2})^2 = \dfrac{2}{3}ma^2$	B1
	$k = \sqrt{\dfrac{I}{M}} = \sqrt{\dfrac{\frac{2}{3}ma^2}{2m}} = \dfrac{a}{\sqrt{3}}$	M1 A1
		(3)
(c)	MI of square about QS = $\dfrac{1}{3}\dfrac{8M}{7}a^2 = \dfrac{8M}{21}a^2$	M1 A1
	MI of square about XY = $\dfrac{8M}{21}a^2 + \dfrac{8M}{7}(\dfrac{a\sqrt{2}}{2})^2$	M1 A1
	$= \dfrac{20Ma^2}{21}$	
	Hence, $I_{PQXYS} = \dfrac{20Ma^2}{21} - \dfrac{1}{6}\dfrac{M}{7}(\dfrac{a}{\sqrt{2}})^2 = \dfrac{79Ma^2}{84}$ PRINTED	M1 A1
		(6)
		16